arXiv:1109.5221v2 [gr-qc] 29 Sep 2011

Nonexistence of quasinormal modes in the extremal BTZ black hole

Yun Soo Myung[1,a], Yong-Wan Kim [1,b], and Young-Jai Park[2,c]

[1]Institute of Basic Science and School of Computer Aided Science,
Inje University, Gimhae 621-749, Korea
[2]Department of Physics and Department of Service Systems Management and Engineering,
Sogang University, Seoul 121-742, Korea

Abstract

We show that quasinormal modes cannot exist in the extremal BTZ black hole. For this purpose, we consider propagations of a minimally coupled scalar and a single massive graviton obtained from the cosmological topologically massive gravity on the extremal BTZ black hole. The would-be quasinormal modes for a scalar and graviton could not exist because it is impossible to make an ingoing flux into the extremal (degenerate) horizon. This is consistent with the argument that there is no propagating dynamics in the self-dual orbifold of AdS_3 which is just the near-horizon limit of the extremal BTZ black hole.

PACS numbers: 04.70.Bw, 04.60.Kz, 04.30.Nk
Keywords: Quasinormal modes; extremal BTZ black hole; topologically massive gravity

[a]ysmyung@inje.ac.kr
[b]ywkim65@gmail.com
[c]yjpark@sogang.ac.kr

1 Introduction

It is well known that Einstein gravity in three dimensions has no propagating degrees of freedom. Massive generalizations of three-dimensional gravity allow propagating degrees of freedom. Topologically massive gravity (TMG) is the famous gravity theory obtained by including a gravitational Chern–Simons term with coupling μ [1, 2]. The model was extended by the addition of a cosmological constant $\Lambda = -1/\ell^2$ to the topologically massive gravity (CTMG) [3]. Since the gravitational Chern–Simons term is odd under parity, the theory shows a single massive propagating degree of freedom of a given helicity, whereas the other helicity mode remains massless. The single massive field is realized as a massive scalar $\varphi = z^{3/2}h_{zz}$ when using the Poincare coordinates $x^{\pm}$ and z covering the AdS_3 spacetimes [4]. However, it was claimed that the massive graviton having negative-energy disappears at the critical point of $\mu\ell = 1$ [5]. This cosmological topological massive gravity at the critical point (CCTMG) may be described by the logarithmic conformal field theory (LCFT) [6, 7] even for the zero central charge $c_L = 0$. Bergshoeff, Hohm, and Townsend recently proposed another massive generalization of Einstein gravity by adding a specific quadratic curvature term to the Einstein-Hilbert action [8, 9]. This term was designed to reproduce the ghost-free Fierz-Pauli action for a massive propagating graviton in the linearized approximation. This gravity theory became known as new massive gravity (NMG). Unlike the TMG, the NMG preserves parity. As a result, the gravitons acquire the same mass for both helicity states, indicating two massive propagating degrees of freedom.

On the other hand, the BTZ black hole [10, 11] as solution to Einstein gravity with Λ is also a black hole solution to CTMG. Its quasinormal modes (QNMs) was calculated in [12, 13] and the CFT approach appeared in [14]. However, this does not necessarily imply that there is no difference in the dynamics of perturbations. It is obvious that the perturbation discriminates between Einstein gravity and CTMG. Moreover, it is worth noting that the QNMs for the tensor perturbation were shown to be the same as those derived by a massive scalar when using the operator method [15]. The asymptotic properties of CTMG were studied in [16, 17]. Recently, it was shown that the non-rotating BTZ black hole is stable for all values of coupling μ against the metric perturbations in CTMG by showing the presence of left-and right-moving normal modes [18]. Very recently, we have checked the stability of the non-rotating BTZ black hole in the NMG by computing quasinormal modes [19]. This indicates that a minimally coupled massive scalar plays a role of the barometer in finding quasinormal modes of the tensor perturbation in the BTZ black hole background.

It is well known that on the contrary to the non-rotating BTZ black hole, the QNMs of the extremal BTZ black hole do not exist for scalar and fermionic perturbations [20]. Similarly,

the QNMs of the massless BTZ black hole did not exist for scalar perturbation [21] and fermionic perturbation [22]. It was suggested that the absence of QNMs of extremal BTZ black hole is closely related to the non-dynamical propagations in the near-horizon limit (a self-dual orbifold of AdS_3) of the extremal BTZ black hole [23, 24, 25].

However, there were two works which report that the QNMs of the extremal BTZ black hole are found for the scalar perturbation [26] and tensor perturbation [27] in CTMG. Let us call these the would-be QNMs.

In this work, we confirm that the would-be QNMs of the extremal BTZ black hole do not exist for scalar and tensor perturbations by showing that there is no ingoing flux onto the extremal horizon. For this purpose, we introduce the Gaussian Normal coordinates (u, v, ρ) to simplify the extremal BTZ geometry.

2 Scalar Propagation

It is known that the extremal BTZ black hole as a solution of three-dimensional Einstein gravity is described by the Schwarzschild coordinates $(t, r, \tilde{\phi})$ as

$$ds^2_{\rm EBTZ} = g_{\mu\nu}dx^\mu dx^\nu = -\Big(\frac{r^2}{\ell^2} - 2\frac{r^2_{ex}}{\ell^2}\Big)dt^2 + \frac{r^2\ell^2}{(r^2 - r^2_{ex})^2}dr^2 - 2\frac{r^2_{ex}}{\ell}dtd\tilde{\phi} + r^2 d\tilde{\phi}^2, \tag{1}$$

whose degenerate horizon $r = r_{ex}$ is determined by $g^{rr} = 0$. For our purpose, we could express it in terms of the Gaussian Normal coordinates (u, v, ρ)

$$ds^2_{\rm EBTZ} = \bar{g}_{\mu\nu}dx^\mu dx^\nu = r^2_{ex}du^2 - \ell^2 e^{2\rho}dudv + \ell^2 d\rho^2, \tag{2}$$

where $u = t/\ell + \tilde{\phi}$, $v = t/\ell - \tilde{\phi}$, and $\ell^2 e^{2\rho} = r^2 - r^2_{ex}$. For simplicity, we choose $\ell = 1$. In this coordinate system, the location of horizon $(r = r_{ex})$ corresponds to $\rho = -\infty$, while the infinity of $r = \infty$ corresponds to $\rho = \infty$. Note that the extremal BTZ spacetime has the asymptotically AdS_3 spacetime.

Now, we are ready to study linear perturbations first for a scalar field in this section, and then for a tensor field in the next section by considering the metric perturbation $h_{\mu\nu}$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \tag{3}$$

The reason why one considers the scalar field first is clearly understood when realizing that the QNMs of a minimally coupled scalar usually provides a prototype of tensor QNMs to any black holes [28, 29]. Then, let us consider a massive scalar field with a mass μ, whose equation of motion is described by

$$(\bar{\nabla}^2 - \mu^2)\Phi = 0, \tag{4}$$

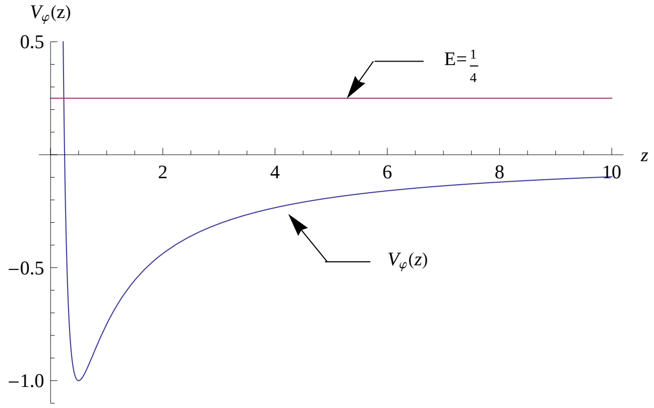

Figure 1: Potential $V_\varphi(z)$ graph as function of z for $\lambda = \mu = 1$.

where the overbar ($^-$) means the extremal BTZ background. Considering the background symmetry, one has the ansatz

$$\Phi(t, r, \phi) = e^{-i\omega t - ik\tilde{\phi}}\varphi(r), \tag{5}$$

which leads to with $h/\bar{h} = (\omega \pm k)/2$

$$\Phi(u, v, \rho) = e^{-ihu - i\bar{h}v}\varphi(\rho). \tag{6}$$

Then, the equation of motion becomes

$$\varphi''(\rho) + 2\varphi'(\rho) + 4e^{-2\rho}(h\bar{h} + \bar{h}^2 r_{ex}^2 e^{-2\rho})\varphi(\rho) - \mu^2\varphi(\rho) = 0, \tag{7}$$

where the prime denotes the differentiation with respect to ρ. If one redefines ρ-coordinate with a new z as

$$z = 2\bar{h} r_{ex} e^{-2\rho}, \tag{8}$$

Eq. (7) leads to the Schrödinger-type equation

$$\frac{d^2\varphi}{dz^2} + \Big[E - V_\varphi(z)\Big]\varphi = 0, \tag{9}$$

where the energy E and potential V_φ are given by

$$E = \frac{1}{4}, \quad V_\varphi(z) = \frac{\mu^2}{4z^2} - \frac{\lambda}{z} \tag{10}$$

with $\lambda = \frac{h}{2r_{ex}}$. Note that the potential depicted in Fig. 1 shows that it grows to infinity as $z \to 0$ ($r \to \infty$), while it approaches slowly zero as $z \to \infty$ ($r \to r_{ex}$).

In order to find the QNMs of a scalar field propagating the extremal BTZ spacetime, one requires the boundary condition: the normalizable mode at $z = 0$ ($r = \infty$) and ingoing mode

at $z = \infty$ $(r = r_{ex})$. Observing the potential and energy leads to the elementary quantum mechanics to determine the wave function at two boundaries. Near $z \sim 0$ $(r \to \infty)$, its normalizable solution for $\mu > 0$ is

$$\varphi_0 \sim z^{\tilde{s}_+}, \quad \tilde{s}_+ = \frac{1 + \sqrt{\mu^2 + 1}}{2} \tag{11}$$

while for $z \to \infty$ $(r \to r_{ex})$, its incoming solution is described by

$$\varphi_\infty \sim e^{\frac{i}{2}z}. \tag{12}$$

On the other hand, the intermediate solution between $z \sim 0$ and $z \to \infty$ is described by taking $\varphi = \varphi_0 \varphi_\infty f_\Phi(z)$, where $f_\Phi(z)$ satisfies the differential equation

$$f''_\Phi(z) + \left(\frac{2\tilde{s}_+ + iz}{z}\right) f'_\Phi(z) + \left(\frac{i\tilde{s}_+ + \lambda}{z}\right) f_\Phi(z) = 0. \tag{13}$$

Redefining $\xi = -iz$, this differential equation becomes

$$\xi \frac{d^2 f_\Phi(\xi)}{d\xi^2} + (2\tilde{s}_+ - \xi)\frac{df_\Phi(\xi)}{d\xi} - (\tilde{s}_+ - i\lambda) f_\Phi(\xi) = 0 \tag{14}$$

whose normalizable solution is determined to be

$$f_\Phi(\xi) \sim F[\tilde{s}_+ - i\lambda, 2\tilde{s}_+; \xi]. \tag{15}$$

Here $F[a, c; \xi]$ is the confluent hypergeometric function [30].

As a result, we have the full solution to Eq. (9) written as

$$\Phi(u, v, z) \sim e^{-ihu - i\bar{h}v} \varphi(z) \tag{16}$$

with

$$\varphi(z) = z^{\tilde{s}_+} e^{\frac{i}{2}z} F[\tilde{s}_+ - i\lambda, 2\tilde{s}_+; \xi]. \tag{17}$$

We will derive the would-be QNMs in the end of the following section because the solution $\Phi(u, v, z)$ has the nearly same form as that of a propagating tensor mode h_{vv}.

3 Tensor Propagation

In this section, we study the linear perturbation for a tensor field in the extremal BTZ background. The action of TMG is given by

$$I_{TMG} = \frac{1}{\kappa^2}\left(I_{EH} + \frac{1}{\mu} I_{CS}\right), \tag{18}$$

with the Einsten-Hilbert (EH) and the gravitational Chern-Simons (CS) action

$$\begin{aligned} I_{EH} &= \int d^3x\sqrt{-g}\left(R+\frac{2}{\ell^2}\right), \\ I_{CS} &= \frac{1}{2}\int d^3x\sqrt{-g}\epsilon^{\lambda\mu\nu}\Gamma^\rho_{\lambda\sigma}\left(\partial_\mu\Gamma^\sigma_{\rho\nu}+\frac{2}{3}\Gamma^\sigma_{\mu\tau}\Gamma^\tau_{\nu\rho}\right). \end{aligned} \tag{19}$$

Here, we denote $\kappa^2 = 16\pi G$, the cosmological constant, $\Lambda = -1/\ell^2$, and a coupling constant, μ. The equation of motion of the TMG is obtained as

$$G_{\mu\nu} + \frac{1}{\mu}C_{\mu\nu} = 0, \tag{20}$$

where the Einstein tensor $G_{\mu\nu}$ and the Cotton tensor $C_{\mu\nu}$ are given by

$$\begin{aligned} G_{\mu\nu} &= R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu}, \\ C_{\mu\nu} &= \epsilon_\mu^{\alpha\beta}\nabla_\alpha\left(R_{\beta\nu}-\frac{1}{4}g_{\beta\nu}R\right), \end{aligned} \tag{21}$$

respectively. As was mentioned in the introduction, the TMG provides a single massive propagating mode.

The perturbed Einstein equation of the TMG under the transverse-traceless (TT) gauge ($\bar{\nabla}^\mu h_{\mu\nu} = 0,\ h^\rho{}_\rho = 0$) leads to the third-order equation [5]

$$\left(\bar{\nabla}^2 + \frac{2}{\ell^2}\right)\left(h_{\mu\nu} + \frac{1}{\mu}\epsilon_\mu{}^{\alpha\beta}\bar{\nabla}_\alpha h_{\beta\nu}\right) = 0, \tag{22}$$

where the overbar ($^-$) means the extremal BTZ background. However, when considering the TT gauge and massive propagation only, it is enough to consider the first-order equation which describes a massive graviton

$$\mu h_{\mu\nu} + \epsilon_\mu{}^{\alpha\beta}\bar{\nabla}_\alpha h_{\beta\nu} = 0 \tag{23}$$

because $\left(\bar{\nabla}^2 + \frac{2}{\ell^2}\right)h_{\mu\nu} = 0$ describes the massless graviton, being gauge-artefact in three dimensions. Starting with six components of a symmetric tensor as

$$h_{\mu\nu} = e^{-ihu-i\bar{h}v}\begin{pmatrix} F_{uu}(\rho) & F_{uv}(\rho) & F_{u\rho}(\rho) \\ F_{uv}(\rho) & F_{vv}(\rho) & F_{v\rho}(\rho) \\ F_{u\rho}(\rho) & F_{v\rho}(\rho) & F_{\rho\rho}(\rho) \end{pmatrix}, \tag{24}$$

we obtain two first-order differential equations from (23)

$$F'_{vv} = -(\mu-1)F_{vv} - i\bar{h}F_{\rho v}, \tag{25}$$

$$F'_{\rho v} = -(\mu+1)F_{\rho v} - i\bar{h}F_{\rho\rho}, \tag{26}$$

and four algebraic relations

$$\begin{aligned}
(\mu+1)F_{\rho u}e^{2\rho} &= 2i(hF_{uv}-\bar{h}F_{uu}),\\
\mu F_{\rho\rho}e^{4\rho} &= 2i(hF_{\rho v}-\bar{h}F_{\rho u})e^{2\rho}+4F_{vv}r_{ex}^2,\\
(\mu-1)F_{\rho v}e^{2\rho} &= 2i(hF_{vv}-\bar{h}F_{uv}),\\
F_{\rho\rho}e^{4\rho} &= 4(r_{ex}^2F_{vv}+e^{2\rho}F_{uv}).
\end{aligned} \tag{27}$$

Then, one may rewrite all other components in terms of F_{vv} and F'_{vv} as

$$F_{v\rho} = \frac{i}{\bar{h}}\left[(\mu-1)F_{vv}+F'_{vv}\right], \tag{28}$$

$$F_{uv} = -\frac{1}{2\bar{h}^2}\left[((\mu-1)^2e^{2\rho}-2h\bar{h})F_{vv}+(\mu-1)e^{2\rho}F'_{vv}\right], \tag{29}$$

$$F_{\rho\rho} = -\frac{2}{\bar{h}^2}\left[((\mu-1)^2-2\bar{h}^2r_{ex}^2e^{-4\rho}-2h\bar{h}e^{-2\rho})F_{vv}+(\mu-1)F'_{vv}\right], \tag{30}$$

$$\begin{aligned}
F_{u\rho} = -\frac{i}{\bar{h}^3}&\left[(\mu(\mu-1)^2e^{2\rho}-2\bar{h}^2r_{ex}^2(\mu-1)e^{-2\rho}+h\bar{h}(1-3\mu))F_{vv}\right.\\
&\left.+(\mu(\mu-1)e^{2\rho}-h\bar{h})F'_{vv}\right],
\end{aligned} \tag{31}$$

$$\begin{aligned}
F_{uu} = \frac{1}{2\bar{h}^4}&\Big[\left(\mu(\mu+1)(\mu-1)^2e^{4\rho}-4\mu^2h\bar{h}e^{2\rho}+2(h^2-r_{ex}^2(\mu^2-1))\bar{h}^2\right)F_{vv}\\
&+\mu\left((\mu^2-1)e^{2\rho}-2h\bar{h}\right)e^{2\rho}F'_{vv}\Big],
\end{aligned} \tag{32}$$

which show that a single propagating mode is F_{vv}.

On the other hand, since the first-order equations are not suitable to derive the QNMs, we need the second-order equation for F_{vv}. Differentiating Eq. (25) with respect to ρ, and replacing F'_{vv}, and $F'_{\rho v}$ again, one arrives at the second-order differential equation [27]

$$F''_{vv}+2F'_{vv}+4(\bar{h}h+\bar{h}^2r_{ex}^2e^{-2\rho})e^{-2\rho}F_{vv}-(\mu-1)(\mu-3)F_{vv}=0, \tag{33}$$

which is the same in Eq. (7) except the last mass term. Hence, we could represent the graviton equation (33) effectively as the massive scalar equation

$$\left[\bar{\nabla}^2-(\mu-1)(\mu-3)\right]F_{vv}=0, \tag{34}$$

which explains clearly why Eq. (4) is considered as a prototype of the tensor-perturbed equation.

Introducing

$$z=2\bar{h}r_{ex}e^{-2\rho}, \tag{35}$$

this equation becomes the Schrödinger-like equation

$$\frac{d^2F_{vv}}{dz^2}+\left[E-V_h(z)\right]F_{vv}=0 \tag{36}$$

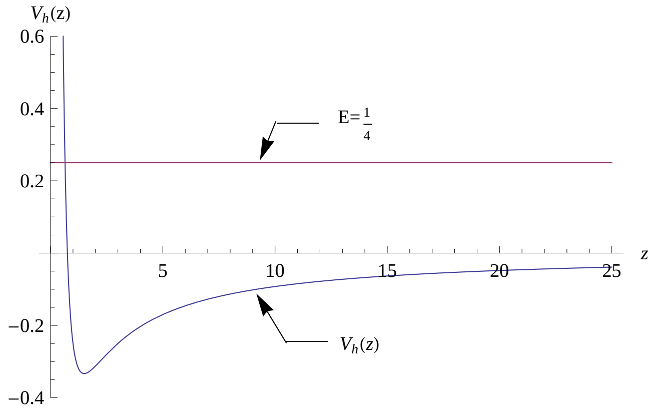

Figure 2: Potential $V_h(z)$ graph as function of z for $\mu = 4$ and $\lambda = 1$.

with the energy E and potential $V_h(z)$

$$E = \frac{1}{4}, \quad V_h(z) = \frac{m^2 - \frac{1}{4}}{z^2} - \frac{\lambda}{z}. \tag{37}$$

Here $m^2 = (\mu/2-1)^2$ and $\lambda = \frac{h}{2r_{ex}}$. Hereafter, we choose $m = \frac{\mu}{2}-1$ without loss of generality. The potential is depicted in Fig. 2. We wish to point out that the shape of potential V_h is very similar to the scalar potential V_φ, which means that their asymptotic forms are the same but the difference is the depth of their potentials. Importantly, their energies are the same. This implies that two fields Φ and h_{vv} may provide the nearly same QNMs if they exist.

In order to find the QNMs of a tensor mode h_{vv}, we require the boundary condition. Near $z \sim 0$ ($r \to \infty$), its normalizable solution for $m > 1/2$ is

$$F^0_{vv} \sim z^{m+\frac{1}{2}} = z^{s_+}, \quad s_+ = \frac{\mu-1}{2}. \tag{38}$$

On the other hand, when $z \to \infty$ ($r \to r_{ex}$), its ingoing solution is

$$F^\infty_{vv} \sim e^{\frac{i}{2}z}. \tag{39}$$

To obtain a full solution in the whole region, we take $F_{vv} = F^0_{vv} F^\infty_{vv} f_h(z)$, and insert it into Eq. (36). Then, the function $f_h(z)$ connecting between the near horizon and the asymptotic infinity satisfies

$$f''_h(z) + \frac{2s_+ + iz}{z} f'_h(z) + \frac{is_+ + \lambda}{z} f_h(z) = 0. \tag{40}$$

Now, defining $\xi = -iz$, we have

$$\xi \frac{d^2 f_h(\xi)}{d\xi^2} + (2s_+ - \xi)\frac{df_h(\xi)}{d\xi} - (s_+ - i\lambda)\, f_h(\xi) = 0, \tag{41}$$

which leads to the same equation as in Eq. (14) when replacing s_+ by $\tilde{s}_+$. Hence, its normalizable solution is given by the confluent hypergeometric function

$$f_h(\xi) \sim F[s_+ - i\lambda, 2s_+, \xi]. \tag{42}$$

Finally, we have the full solution as

$$h_{vv} \sim e^{-ihu - i\bar{h}v} F_{vv}(z) \tag{43}$$

with

$$F_{vv}(z) = z^{s_+} e^{\frac{i}{2}z} F[s_+ - i\lambda, 2s_+, \xi] \tag{44}$$

which indicates the normalizable solution near $z \sim 0$ and the ingoing mode at $z \to \infty$.

Before making a further analysis, we observe the useful property for the complex conjugate of the confluent hypergeometric function as

$$F^*[a, c; \xi] = F[a^*, c; -\xi], \tag{45}$$

where $a = s_+ - i\lambda$ and $c = 2s_+$. Together with the Kummer's transformation of $F[a, c; \xi] = e^{\xi} F[c - a, c; -\xi]$, Eq. (45) implies that the product of $e^{\frac{i}{2}z}$ and $F[a, c; \xi]$ is real as

$$\left[e^{\frac{i}{2}z} F[a, c; \xi]\right]^* = e^{\frac{i}{2}z} F[a, c; \xi]. \tag{46}$$

Now, we are in a position to calculate the radial flux defined by

$$\mathcal{F}_\phi = 2\frac{2\pi}{i}\sqrt{-g}[\phi^* \partial_\rho \phi - \phi \partial_\rho \phi^*] \to 8\pi i \bar{h} r_{ex}[\phi^* \partial_z \phi - \phi \partial_z \phi^*] \tag{47}$$

for a proper mode solution $\phi \in \{\varphi(z), F_{vv}(z)\}$.

Firstly, we calculate the flux near $z \sim 0$ ($r \to \infty$). For $z \to 0$, it is clear that $F[s_+ - i\lambda, 2s_+, \xi] \to 1$ and $e^{\frac{i}{2}z} \to 1$. Thus, the full normalizable solution reduces to

$$F_{vv} \to F^0_{vv} \sim z^{s_+}, \tag{48}$$

which makes the flux zero ($\mathcal{F}|_{z\to 0} = 0$) because F^0_{vv} is real. This is consistent with the Dirichlet boundary condition at infinity of AdS_3 spacetimes.

Secondly, we compute the ingoing flux near the extremal horizon at $z \to \infty$ ($r \to r_{ex}$). Since $e^{\frac{i}{2}z} F[s_+ - i\lambda, 2s_+, \xi]$ is real, the flux of $\mathcal{F}|_{z\to\infty}$ is simply zero. On the other hand, if one uses $F^\infty_{vv} \sim e^{\frac{i}{2}z}$ to compute the ingoing flux, one has the non-zero flux of $\mathcal{F}|_{z\to\infty} = -8\pi \bar{h} r_{ex}$. This contradiction arises because we do not develop an explicit form of wave function in the near-horizon geometry of the extremal BTZ black hole. In order to obtain the desired

wave function in the near-horizon region, we use the expansion formula of the confluent hypergeometric function near the extremal horizon of $z \to \infty$. For large $|\xi|$, one has [30]

$$F[a,c;\xi] = \frac{\Gamma(c)}{\Gamma(c-a)} e^{\pm i\pi a}\xi^{-a} + \frac{\Gamma(c)}{\Gamma(a)} e^{\xi}\xi^{a-c}, \tag{49}$$

where the upper sign is for $-\pi/2 < \arg(\xi) < 3\pi/2$ and the lower sign is for $-3\pi/2 < \arg(\xi) \leq -\pi/2$. Since we use $\xi = -iz$ here, we take the lower sign in the asymptotic expansion which becomes explicitly

$$F[a,c;\xi] = \frac{\Gamma(c)}{\Gamma(c-a)} |\xi|^{-a} e^{-i\pi a/2} + \frac{\Gamma(c)}{\Gamma(a)} e^{\xi}|\xi|^{a-c} e^{i\pi(c-a)/2}. \tag{50}$$

As a result, one finds that the explicit form of wave function

$$\begin{aligned} e^{\frac{i}{2}z}F[a,c;\xi]|_{z\to\infty} &\sim \Bigg[\frac{\Gamma(2s_+)}{\Gamma(s_+ + i\lambda)} e^{-\frac{\pi}{2}\lambda} e^{i(\frac{z}{2}+\lambda\ln z - \frac{\pi}{2}s_+)} \\ &\qquad + \frac{\Gamma(2s_+)}{\Gamma(s_+ - i\lambda)} e^{-\frac{\pi}{2}\lambda} e^{-i(\frac{z}{2}+\lambda\ln z - \frac{\pi}{2}s_+)}\Bigg] \\ &\equiv F^{in}_{vv} + F^{out}_{vv} \end{aligned} \tag{51}$$

in the near-horizon region of the extremal BTZ black hole. We note that the first term is the ingoing mode ($\rightarrow$), while the second term is the outgoing mode ($\leftarrow$) near $z = \infty$ ($r = r_{ex}$). Importantly, we confirm that $e^{\frac{i}{2}z}F[a,c;\xi]|_{z\to\infty}$ is real because of $[F^{in}_{vv}]^* = F^{out}_{vv}$. In order to obtain the QNMs, the wave function should be purely ingoing mode near the extremal horizon. This may be done by requiring $s_+ - i\lambda = -n$, $(n = 1, 2, 3, \cdots)$, which amounts to taking the ingoing flux without outgoing flux. In this case, we may obtain the would-be QNMs of a tensor mode h_{vv} from the condition of $s_+ - i\lambda = -n$ with $\lambda = h/2r_{ex}$ and $s_+ = (\mu - 1)/2$

$$\omega_h = -k - i4r_{ex}\Big(n + s_+\Big), \tag{52}$$

which was exactly the same QNMs found in [27]. However, since the corresponding ingoing-radial flux* is zero when requiring the condition of the no-outgoing mode ($s_+ - i\lambda = -n$) as

$$\begin{aligned} \mathcal{F}^{\rm in}_h(z\to\infty) &= 8\pi i\bar{h} r_{ex}[F^{in*}_{vv}\partial_z F^{in}_{vv} - F^{in}_{vv}\partial_z F^{in*}_{vv}] \\ &= -8\pi\bar{h} r_{ex} e^{-\pi\lambda}\left[\frac{\Gamma(2s_+)}{\Gamma(s_+ - i\lambda)}\frac{\Gamma(2s_+)}{\Gamma(s_+ + i\lambda)}\right] = 0, \end{aligned} \tag{53}$$

*More precisely, we have a factor of $\left(1 + \frac{2\lambda}{z}\right)$ in the front of this expression. However, in the limit of $z \to \infty$, this reduces to 1.

it concludes that there exist no QNMs for the tensor perturbation in the extremal BTZ background. Here, $\Gamma(-n) = \infty$, and the minus sign means that it is the ingoing flux near $z \to \infty$.

The same thing happens to the massive scalar mode by replacing s_+ by $\tilde{s}_+$. We may take the would-be QNMs of a scalar mode Φ

$$\omega_\Phi = -k - i4\pi T_L\Big(n + \tilde{s}_+\Big), \tag{54}$$

which is exactly the same QNMs found in [26] with the left-temperature $T_L = r_{ex}/\pi$ and $\tilde{s}_+ = (1 + \sqrt{\mu^2 + 1})/2$. However, its ingoing flux is zero when requiring $\tilde{s}_+ - i\lambda = -n$

$$\begin{aligned}\mathcal{F}^{\rm in}_\Phi(z \to \infty) &\sim 8\pi i\bar{h} r_{ex}[\varphi^{in*}\partial_z\varphi^{in} - \varphi^{in}\partial_z\varphi^{in*}] \\ &= -8\pi\bar{h} r_{ex} e^{-\pi\lambda}\left[\frac{\Gamma(2\tilde{s}_+)}{\Gamma(\tilde{s}_+ - i\lambda)}\frac{\Gamma(2\tilde{s}_+)}{\Gamma(\tilde{s}_+ + i\lambda)}\right] = 0,\end{aligned} \tag{55}$$

which implies that there is no QNMs of scalar perturbation, too. This consists with the previous works [20, 21] for a massive scalar propagation. Also, the absence of QNMs is consistent with the argument that there is no propagating dynamics in the self-dual orbifold of AdS_3, which is just the near-horizon limit of the extremal BTZ black hole [23, 24, 25].

4 Discussions

In this work we have shown that the would-be quasinormal modes of a massive scalar and a single massive graviton do not exist in the extremal BTZ black hole. This shows the contradiction to the previous results on the scalar propagation [26] and a graviton propagation in the TMG [27].

It was believed that the extremal BTZ black hole is a non-dissipative system because its thermodynamic quantities are characterized by the zero temperature and heat capacity $T_H = C_J = 0$, but the non-zero entropy $S_{BH} = \frac{\pi r_{ex}}{2G}$ [$S_{BH} = \frac{\pi r_{ex}}{2G}(1 + \frac{1}{\mu})$ for the TMG]. Actually, two propagating equations provide the nearly same Schrödinger-type equations (9) and (36) with the same energy $E = 1/4$. If the Schrödinger operator $\mathcal{L} = -d^2/dz^2 + V(z)$ is self-adjoint ($\mathcal{L}^\dagger = \mathcal{L}$), its eigenvalue is real upon imposing the AdS_3-boundary condition. In this case, there is no information loss via either evaporation or absorption process and thus, the unitarity is preserved. This is consistent with the picture that the extremal BTZ black hole is a final remnant, which never evaporates and absorbs any radiations. If the quasinormal modes (complex ω) are found from the black hole in the AdS_3 spacetime, the black hole is regarded as a dissipative system. Therefore, the ingoing flux is not zero at the horizon and the flux is zero at the infinity. However, the extremal black hole including

the massless BTZ black hole belongs to the non-dissipate system, contrary to the dissipative system of the non-extremal BTZ black hole including the non-rotating BTZ black hole. Hence it is reasonable to consider that quasinormal modes could not obtained from the extremal BTZ black hole [31]. From (43), the normal mode solution of $h_{vv} \sim e^{-i\omega t - ik\tilde{\phi}} F_{vv}(z)$ with real ω is allowed only, showing that the extremal BTZ black hole is stable against the external perturbations.

Initially, the BTZ black hole could be holographically described by a dual CFT with both left- and right-moving temperatures [14]. Since the extremal BTZ black hole has the zero Hawking temperature and zero right-temperature, it was believed that one sector of the CFT is frozen, while the other sector survives with $T_L = r_{ex}/\pi$. In this case, some people may consider that the would-be QNMs of extremal BTZ black hole correspond to the operators perturbing the thermal equilibrium in the dual chiral CFT. However, it was suggested that there is no propagating dynamics in the AdS_2 base of the self-dual orbifold of AdS_3 ($\mathrm{AdS}_2 \times S^1$) which is just the near-horizon limit of the extremal BTZ black hole [23]. The near-horizon limit of the extremal black hole is truly dual to the discrete light-cone quantization (DLCQ) of a non-chiral (ordinary) CFT. The kinematics of DLCQ implies that in a consistent quantum theory of gravity around the extremal BTZ black hole, there is no dynamics in the AdS_2. In other words, the description of extremal BTZ black hole in terms of an AdS_2 throat requires asymptotic boundary conditions eliminating AdS_2 excitations. How would the TMG around the extremal BTZ black hole require the absence of a massive graviton h_{vv} including a massive scalar Φ in the AdS_2 base of $\mathrm{AdS}_2 \times S^1$? This may be because any fluctuations would cause the space to fragment, leading to the appearance of multiple boundaries to the spacetimes [32].

Finally, if one finds the retarded correlation function in the DLCQ theory, according to the AdS/CFT dictionary one can confirm the absence of QNMs of the extremal BTZ black hole because the QNMs can read off from the location of the poles of the retarded correlation function of the corresponding perturbations in the dual CFT [25]. For the massless BTZ black hole, it was confirmed from the CFT side [22] that there is no QNMs of a massive scalar propagation [21].

Acknowledgement

Two of us (Y. S. Myung and Y.-W. Kim) were supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (No.2011-0027293). Y.-J. Park was supported by the Korea Science and Engineering Foundation (KOSEF) grant funded by the Korea government (MEST) through WCU Program (No. R31-20002).

References

[1] S. Deser, R. Jackiw and S. Templeton, Phys. Rev. Lett. **48**, 975 (1982).

[2] S. Deser, R. Jackiw and S. Templeton, Annals Phys. **140**, 372 (1982) [Erratum-ibid. **185**, 406 (1988)] [Annals Phys. **281**, 409 (2000)].

[3] S. Deser, in *Quantum Theory of Gravity*, edited by S.M. Christensen (Adam Hilger, London, 1984).

[4] S. Carlip, S. Deser, A. Waldron and D. K. Wise, Class. Quant. Grav. **26**, 075008 (2009) [arXiv:0803.3998 [hep-th]].

[5] W. Li, W. Song and A. Strominger, JHEP **0804**, 082 (2008) [arXiv:0801.4566 [hep-th]].

[6] D. Grumiller and N. Johansson, JHEP **0807**, 134 (2008) [arXiv:0805.2610 [hep-th]].

[7] Y. S. Myung, Phys. Lett. B **670**, 220 (2008) [arXiv:0808.1942 [hep-th]].

[8] E. A. Bergshoeff, O. Hohm and P. K. Townsend, Phys. Rev. Lett. **102**, 201301 (2009) [arXiv:0901.1766 [hep-th]].

[9] E. A. Bergshoeff, O. Hohm and P. K. Townsend, Phys. Rev. D **79**, 124042 (2009) [arXiv:0905.1259 [hep-th]].

[10] M. Banados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. **69**, 1849 (1992) [arXiv:hep-th/9204099].

[11] M. Banados, M. Henneaux, C. Teitelboim, J. Zanelli, Phys. Rev. **D48**, 1506-1525 (1993). [gr-qc/9302012].

[12] J. S. F. Chan, R. B. Mann, Phys. Rev. **D55**, 7546 (1997). [gr-qc/9612026].

[13] V. Cardoso, J. P. S. Lemos, Phys. Rev. **D63**, 124015 (2001). [gr-qc/0101052].

[14] D. Birmingham, I. Sachs, S. N. Solodukhin, Phys. Rev. Lett. **88**, 151301 (2002) [hep-th/0112055].

[15] I. Sachs and S. N. Solodukhin, JHEP **0808**, 003 (2008) [arXiv:0806.1788 [hep-th]].

[16] M. Henneaux, C. Martinez, R. Troncoso, Phys. Rev. **D79**, 081502 (2009). [arXiv:0901.2874 [hep-th]].

[17] M. Henneaux, C. Martinez, R. Troncoso, Phys. Rev. **D82**, 064038 (2010). [arXiv:1006.0273 [hep-th]].

[18] D. Birmingham, S. Mokhtari and I. Sachs, Phys. Rev. D **82**, 124059 (2010) [arXiv:1006.5524 [hep-th]].

[19] Y. S. Myung, Y. -W. Kim, T. Moon, Y. -J. Park, Phys. Rev. **D84**, 024044 (2011) [arXiv:1105.4205 [hep-th]].

[20] J. Crisostomo, S. Lepe, J. Saavedra, Class. Quant. Grav. **21**, 2801-2810 (2004) [hep-th/0402048].

[21] Y. S. Myung, H. W. Lee, Mod. Phys. Lett. **A21**, 1737-1748 (2006) [hep-th/0506031].

[22] X. -P. Rao, B. Wang, G. -H. Yang, Phys. Lett. **B649**, 472-477 (2007) [arXiv:0712.0645 [gr-qc]].

[23] V. Balasubramanian, J. de Boer, M. M. Sheikh-Jabbari and J. Simon, JHEP **1002**, 017 (2010) [arXiv:0906.3272 [hep-th]].

[24] V. Balasubramanian, J. Parsons, S. F. Ross, Class. Quant. Grav. **28**, 045004 (2011) [arXiv:1011.1803 [hep-th]].

[25] J. de Boer, M. M. Sheikh-Jabbari, J. Simon, Class. Quant. Grav. **28**, 175012 (2011) [arXiv:1011.1897 [hep-th]].

[26] B. Chen, J. -j. Zhang, Phys. Lett. **B699**, 204-210 (2011) [arXiv:1012.2219 [hep-th]].

[27] H. R. Afshar, M. Alishahiha, A. E. Mosaffa, JHEP **1008**, 081 (2010) [arXiv:1006.4468 [hep-th]].

[28] E. Berti, V. Cardoso, A. O. Starinets, Class. Quant. Grav. **26**, 163001 (2009). [arXiv:0905.2975 [gr-qc]].

[29] R. A. Konoplya, A. Zhidenko, Rev. Mod. Phys. **83** , 793 (2011) [arXiv:1102.4014 [gr-qc]].

[30] M. Abramowitz and I. Stegan, *Handbook of Mathematical Functions* (Academic Press, New York, 1966).

[31] A. Lopez-Ortega, Int. J. Mod. Phys. **D19**, 1973-1985 (2010) [arXiv:1006.5429 [gr-qc]].

[32] J. M. Maldacena, J. Michelson, A. Strominger, JHEP **9902**, 011 (1999) [hep-th/9812073].